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Liquid flow in tubes. IV

The transition process and turbulent flow related to tube diameter and microscopic surface properties

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With 9 figures in the text

ABSTRACT

Studies previously reported on the transition process have been carried further by variation of tube diameter and surface finish.

The experiments indicate the influence of tube diameter and microscopic surface roughness on certain transition quantities. Thus, the transition Reynolds number, $\underline{R}$ (= the lowest Reynolds number at which self-preserving turbulent slugs ever appear), increased from about 1800 to 2200 (ca. 20 % increase) when the tube diameter increased from about 4 mm to 24 mm (ca. 600 % increase). A change of the microscopic surface roughness by sand-blasting of the tubes did not influence the $\underline{R}$ values nor the relative rear velocity of the turbulent slugs. However, it did increase their relative front velocity and also decreased the critical Reynolds number, R_k (= the Reynolds number at which the turbulent slugs begin to split or expand during their downstream travel).

Hence, in considering present and previous experimental findings, it seems that the transition process and turbulence maintenance mechanism involve a complicated pattern of factors which at present cannot be separated from one another.

Introduction

In the previous paper of this series (Liquid flow in tubes. III, Lindgren, 1959*c*) report was made of indications that the Reynolds number would not be a sufficient similarity parameter and that some other physical properties of the flowing liquid than merely viscosity may have an influence on the flow characteristics. Also, a transition quantity, $(U_R/U)_{\max}$, which does not formally include any physical quantities other than the mean flow velocity and the turbulent slug velocity, was defined and reported. The present study was undertaken as a natural continuation of those experiments with the aim of investigating whether tube dimensions and microscopic surface properties also would influence the transition pattern as did some "structural" properties of the flowing liquid according to the previous study. Neither possibility is predicted by the theories of classical hydrodynamics.

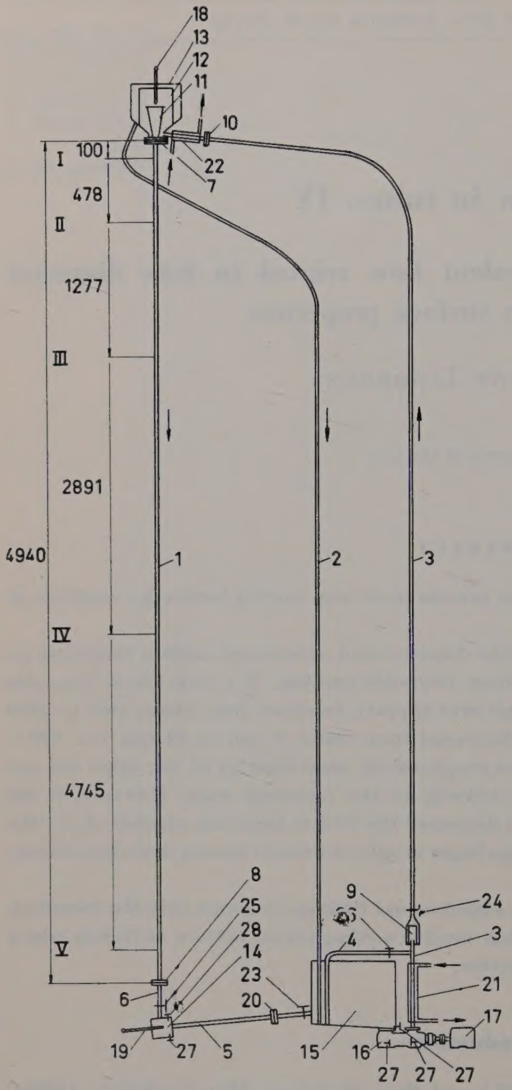

Fig. 1. Arrangement of the experimental flow apparatus

1. Experiment tube
2. Overflow return pipe (PVC plastics)
3. Feeding pipe line (PCV plastics)
4. Short circuit plexiglasspipe
5. 2 measuring pipes
6. Joint hose (PCV plastics)
7. Inlet arrangement from the levelling tank to the experiment tube (see Fig. 2)
8. Coupling flanches
9. Glass wool sieve
10. Glass-fiber-fabric sieve
11. Outlet trumpet from the levelling tank
12. Levelling tank
13. Covering container
14. Calming tank
15. Supply container
16. Circulation pump
17. Electric motor

18, 19. Thermometers

20. 2 housings with measuring orifice plates

21, 22. Heat exchangers

23. 2 regulation cocks

24–26. Stop cocks

27. Tap cocks

28. Bleed cock

Experimental arrangement

The experimental apparatus was the same as that used in previous experiments reported in this series (Lindgren, 1959*a*, *b* and *c*). It is shown in Fig. 1.

Five extruded, jointless plexiglass tubes of 4.94 m length and of about the same surface finish were secured for the experiments. Local diameter values of the tubes were obtained by the process of step-wise filling and emptying the tubes in about 0.3 m lengths. All mean tube diameters, d, thus measured fell within the limits:

$$d = 3.84 \pm 0.06;\ 6.13 \pm 0.04;\ 9.80 \pm 0.04;\ 16.05 \pm 0.07;\ 23.72 \pm 0.06 \text{ mm}.$$

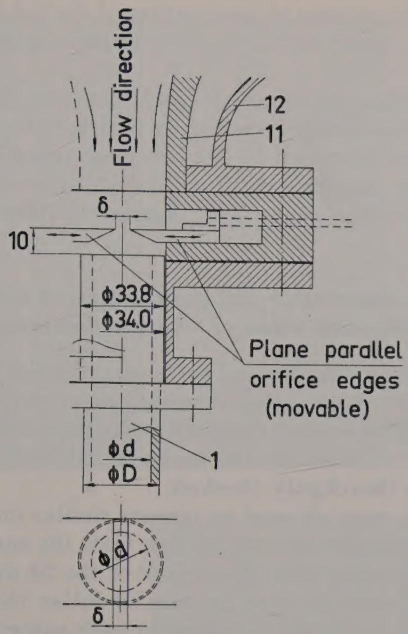

Fig. 2. Arrangement of the experiment tube inlet. 1, experiment tube; 11, outlet trumpet from the levelling tank; 12, levelling tank: d = inner tube diameter, D = outer tube diameter. During the present set of experiments the disturbance level of the inlet flow was varied by variation of the width, δ, of the slit between the orifice edges. (Above the coupling flange the outer diameter of the experiment tube is increased to 33.8 mm in order to avoid a "dead" liquid volume.)

(The 23.72 mm tube is identical with that used in the experiments reported in the previous paper in this series although a misprint there gave the diameter as 23.71 mm.) The roundness of the tubes has been checked at four cross-sections at the ends of the tubes from which the experiment tubes were cut. At these locations the deviation from the mean diameter in no case exceeded 0.05 mm.

For the convenience of the reader the inlet arrangement for the tubes, Fig. 2, is shown again. The inlet disturbance level of the flow was varied by changing the width and symmetry of the opening between the two movable parallel edges.

The same stream bi-refringence technique as before has been applied also in the present experiments using a White Hector bentonite sol (of distilled water) of 1.7 ‰ concentration. The concentration value has been obtained with an accuracy of about 1 %. This sol is not prepared from the same parent solution as the series of sols used in the experiments of the preceding paper of this series (Lindgren, 1959*c*) but was prepared from the same parent solution that was made about eight years ago and which was used in experiments reported earlier (Lindgren, 1954 *a*, *b*, and 1957). This is an important fact since it gives a demonstration of the stability and consistency of the sols used, and in addition provides a test of the reproducibility and reliability of the experimental results so far obtained by this method.

For a description of the process of preparation of the sol and the determination of its viscosity, reference is made to the above-mentioned papers (Lindgren, 1957, pp. 16–17; 1959*c*, p. 112).

The process of formation, development and transport of turbulent slugs in the flow of the sol was simultaneously recorded by means of five optic-electronic devices placed along the tubes as described in the last paper mentioned above. That paper also describes the method used for measuring the rate of flow by means of orifice plates and strain-gauge pressure drop transducers (Lindgren, 1959*c*, pp. 103–104).

Observations on the appearance and propagation of turbulent slugs in flow through tubes of various diameters

As before, the observations are limited to those processes which do not necessitate an analysis of quantitative relationships between shear stress in the flow at the location of the recording light beam and transmitted signals. The only stipulation is that differences in the state of flow should be recorded with sufficient sensitivity, a condition that has been thoroughly checked.

The experiment tubes were planned to possess similar inner surface structure, an aim that was fairly well reached, as can be seen from the surface micrographs of Fig. 3 A. It may be noted, however, that the surface of the 24 mm tube is smoother than that of the 16 mm tube, which in turn appears smoother than those of the 10, 6 and 4 mm tubes, which seem to be more similar to each other. On the other hand, the height of the surface roughness elements in no case seems to exceed about 3/1000 mm.

In view of the above data some confidence may be put in the experimental results obtained as reported in the plots of Fig. 4,[1] which shows the relative velocities of the front, U_F/U, and the rear, U_R/U, of the slugs in relation to the Reynolds number $R=Ud/\nu$ (U_F=front velocity; U_R=rear velocity of the turbulent slugs; U=mean flow velocity; d=tube diameter; ν=kinematic viscosity of the flowing liquid) for flow through the tubes reported above.

The plots indicate an increase of the transition Reynolds number $\underline{R}$ (=the smallest Reynolds number at which self-preserving tubulent slugs are observed ever to appear) with increasing tube diameter, d. We also note a corresponding decrease of the transition quantity $(U_R/U)_{\max}$. It is of special importance to note that the plots of U_R/U, and consequently also the transition quantities $\underline{R}$ and $(U_R/U)_{\max}$, appear to vary with R along one and the same curve without any noticeable shift for varying tube diameter, d.

The plots of Fig. 4 also reveal an increase of the critical Reynolds number R_k (=the Reynolds number at which the slugs begin to elongate themselves during their tube-passage) as the tube-diameter increases, with corresponding shifts of the plots for the relative front velocity, U_F/U, of the turbulent slugs. In agreement with previous experiments, the recorded data for the front of the slugs are more irregularly spread than those for their rear.

The systematic variation of the transition quantities in relation to the tube diameter could possibly be traced to the higher velocity gradients of the flowing liquid in the case of the smaller tubes as compared to the larger ones at the same Reynolds number. It may be questioned, however, in what manner this change of velocity

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[1] It should be noted that each plotted velocity value in the diagrams of this paper represents the mean value from the passage of about six slugs.

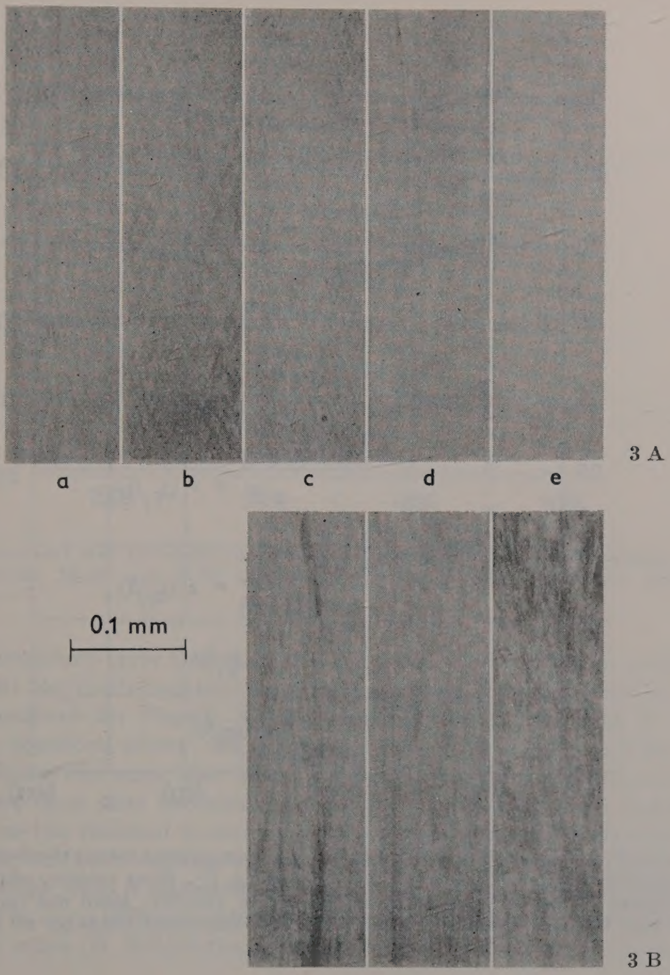

Fig. 3. Surface micrographs of the experiment tubes. (A) before sandblasting; (B) after sandblasting of the three larger tubes.

The shadowed micrographs do not permit an accurate determination of the surface roughnesses, but from the scale of magnification it may be concluded that the height of the roughness elements before sand-blasting in no case seems to exceed about 3/1000 mm.

The micrographs are of the (*a*) 3.84, (*b*) 6.13, (*c*) 9.80, (*d*) 16.05 and (*e*) 23.72 mm tubes.

Note that the surfaces of the 16 and 24 mm tubes, especially the 24 mm tube, appear considerably smoother than those of the smaller tubes which approximately show about the same surface texture.

The surfaces of the three larger tubes show a distinct increase of the surface roughness as due to the sand-blasting, and evidently the surface of the 24 mm tube shows the largest roughness elements while those of the 10 mm tube surface are smaller but more distinct.

The difference of surface roughness before and after sand-blasting is largest for the surface of the 24 mm tube and smallest for that of the 10 mm tube. This state of affairs correlates to the experimental results according to which the sand-blasting influenced certain features of the transition process most for flows through the 24 mm tube and least for flows through the 10 mm tube.

It is important to note that the height of the surface roughness elements of the three tubes after the sand-blasting still seems to be at least an order of magnitude smaller than the thickness of the assumed viscous layer for self-preserving turbulent tube flow within the actual Reynolds number region. (In no case do the roughness elements appear to exceed a height of 2/100 mm.)

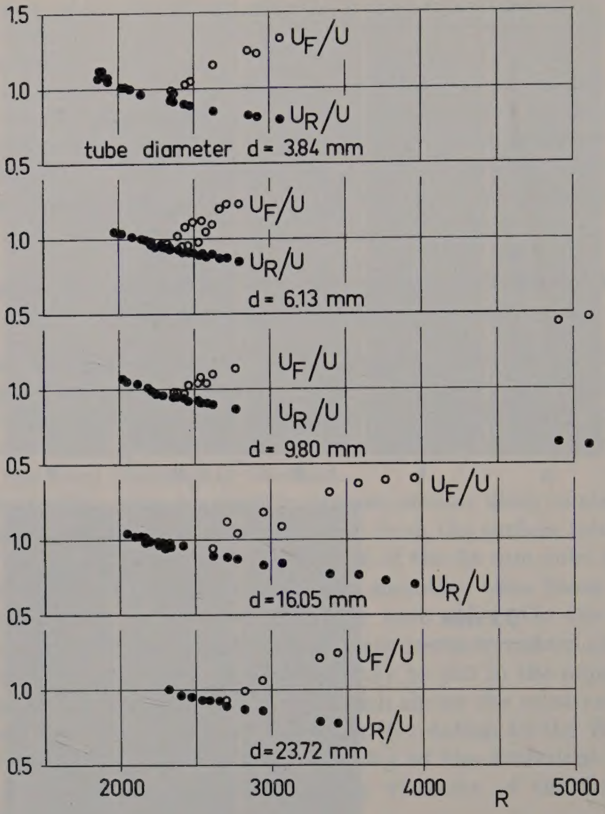

Fig. 4. Relative front and rear velocity of turbulent slugs plotted versus the Reynolds number for flows through the experiment tubes before sandblasting. U_F, front velocity of the turbulent slug. U_R, rear velocity of the turbulent slug. U, mean flow velocity. Each dot represents the mean value of at least 6 experimental determinations. This statement holds for all plots given in this paper.

gradients could influence the transition pattern. Two possibilities suggest themselves. Either, some internal structure of the liquid is influenced by the rate of shear resulting in a change of the dynamic properties of the liquid in itself (or relative to the properties of the bounding walls); or the influence, perhaps, could be due to purely mechanical effects of the bounding walls relative to variations of the flow velocity.

If the case were that purely mechanical effects of the bounding walls were responsible for the reported relationships of the transition pattern one would expect this to be disclosed by changing the surface texture of the experiment tubes, repeating the experiments and comparing the transition plots before and after treatment of the tube surfaces.

It should be noted, however, that this discussion concerns effects inconsistent with generally adopted concepts of boundary-layer theory. For instance, the surface texture of the experiment tubes for the plots in Fig. 4 is built up by roughness elements that are several orders of magnitude smaller than those which are claimed necessary

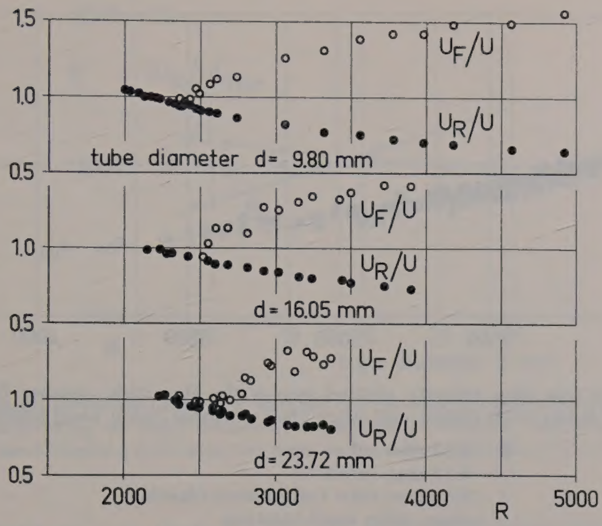

Fig. 5. Relative front and rear turbulent slug velocity in relation to Reynolds number for flows through the 9.80, 16.05 and 23.72 mm tubes after sand-blasting of their surfaces.

by the classical boundary-layer theory in order that the walls should be hydraulically smooth, within the Reynolds number region in question, and not influence the transition process (*Handbuch der Physik*, VIII/1, 1959, pp. 440–450). Thus, in order to perform the test described above, the surface texture of the tubes must be changed so that the roughness elements, also after the change, should be at least an order of magnitude smaller than that necessary to consider the tube surfaces hydraulically smooth. Otherwise the classical boundary-layer theory could be used to claim some influence from the surface roughness on the transition process and on the behaviour of the turbulent slugs, even if standard textbooks in fluid dynamics state that surface roughness should have no influence whatsoever on the transition between laminar and turbulent flow in tubes (H. Schlichting, 1951, p. 380; J. Knudsen & D. Katz, 1958, p. 174).

Effects from microscopic surface roughness on the transition process

In view of the above discussion, the three larger tubes were sand-blasted for some seconds by means of fine quartz-sand. The micrographs of Fig. 3 B show the appearance of the surfaces after the sand-blasting. We note a distinct increase of the surface roughness although the heights of the surface roughness elements do not seem to exceed 2/100 mm, which is at least an order of magnitude smaller in size than the thickness of an assumed viscous layer even for the smallest tube (9.80 mm diameter) for which it should be about 0.2 mm for fully developed turbulent flow at a Reynolds number of about $R \approx 3000$.

Fig. 5 shows the plots of the experimental results for flow through the three sand-blasted tubes. By comparison with the plots of Fig. 4 we find that the transition Reynolds number, $\underline{R}$, does not seem to be influenced by the change of microscopic

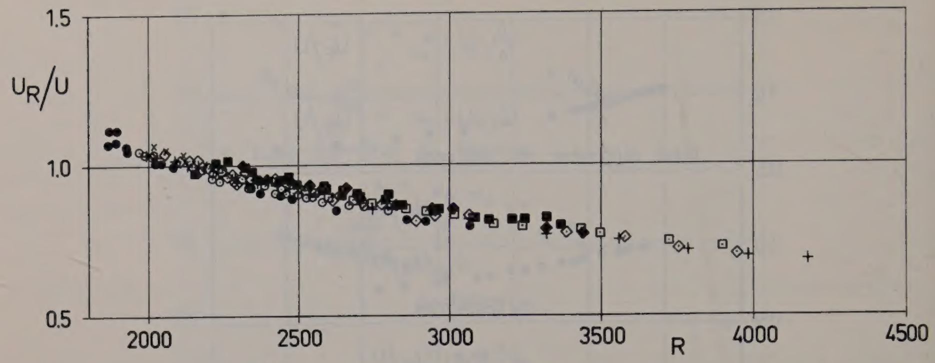

Fig. 6. The relative rear slug velocity plotted versus the Reynolds number for flows through all the experiment tubes both before and after their surfaces being sand-blasted.

● = 3.84 mm tube
○ = 6.13 mm tube
× = 9.80 mm tube before sand-blasting
+ = same, after sand-blasting
◇ = 16.05 mm tube before sand-blasting
□ = same, after sand-blasting
◆ = 23.72 mm tube before sand-blasting
■ = same, after sand-blasting

surface roughness, nor does it seem to have had any bearing on the propagation velocity of the rear of the slugs. On the other hand, it appears that the change of roughness has had some influence on the front velocity of the slugs and also seems to have caused some decrease of the critical Reynolds number, R_k.

Summary of observations

The rear velocity of the slugs

For the purpose of analysing the experimental observations, all velocity data referring to the rear of the slugs for flows through all experiment tubes, with and without sand-blasted surfaces, have been collected in the plot of Fig. 6. It confirms the impression from the plots of Fig. 5 that no influence on the rear velocity of the slugs can be traced from the change of roughness. Nor does the tube diameter seem to have any specific bearing in those respects, although perhaps a slight tendency towards increasing relative rear velocity of the slugs may be interpreted as due to increasing tube diameter. This drift of the U_R/U values, however, lies approximately within the limits of accuracy of the experiments and is therefore by no means an observation that, at present, should be assigned any significance.

In detail, we observe that the least $\underline{R}$-values obtained from flows through the different tubes refer to the 16 mm tube before sand-blasting and to the 10 and 24 mm tubes after sand-blasting. The least $\underline{R}$-values obtained, whether sand-blasted tubes or not, are plotted in Fig. 7 in relation to the tube diameter, d. Note, that corresponding $(U_R/U)_{\max}$ values may be read from the single plot of Fig. 6, independent of tube diameter. These values are also plotted in Fig. 7 in relation to the tube diameter.

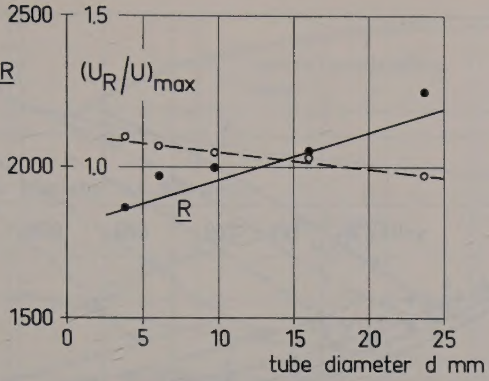

Fig. 7. The transition Reynolds number $\underline{R}$ and the corresponding maximum relative slug velocity, $(U_R/U)_{\max}$, as depending on tube diameter, regardless of whether surfaces were sand-blasted or not, since the sand-blasting here does not seem to have had any influence on these experimental data.

It is remarkable to note that, in spite of a distinct increase of the $\underline{R}$-value with increasing diameter, d, there is no noticeable change of U_R/U with varying d for the same Reynolds number flow.

Another peculiarity is that the least $\underline{R}$-value obtained for flow of the present 1.7 ‰ sol through the 24 mm tube is $\underline{R} \approx 2320$ before sand-blasting and $\underline{R} \approx 2230$ after, while it was $\underline{R} \approx 2380$ for the 1.21 ‰ sol and $\underline{R} \approx 2560$ for the 2.40 ‰ sol flowing through the same 24 mm tube (not sand-blasted) in experiments reported in the previous study of this series (Lindgren, 1959*c*, Fig. 4, p. 107). This means that, for full agreement between the previous and the present experiments, an $\underline{R}$-value is required of about 2460 for flow of the 1.7 ‰ sol, instead of the value $\underline{R} \approx 2230$–$2320$ obtained.

This state of affairs becomes quite interesting if we plot the data now obtained for flow through the 24 mm tube within the plots of the previous Fig. 4, mentioned above. This has been done in Fig. 8, and we should also remember here that the 1.2 and 2.4 ‰ bentonite sols were diluted from a different parent sol than that from which the 1.7 ‰ sol used at present was diluted, as has already been mentioned (p. 451).

From the plot in Fig. 8 we infer that the U_R/U values obtained at present are about properly situated between the corresponding values for flow of the 1.2 and 2.4 ‰ bentonite sols, but that they extend smoothly the region of existence of turbulent slugs towards lower Reynolds numbers with an appropriate increase of incident U_R/U values. In this connection it is also noted that the flow of the 1.7 ‰ sol appears to have higher relative front velocity of the slugs for the same Reynolds numbers and also to have a lower critical Reynolds number than the flows of the 1.2 and 2.4 ‰ sols.

There seems to be but one possible explanation of the extension of the region of existence for the turbulent slugs and the increase of the relative front velocity of the slugs for flows of the 1.7 ‰ sol in comparison with the other two. This is that it should be due to some physical—say structural—differences between the two parent suspensions, since everything else is identical in the performance of the experiments. Such differences may be due to some minor variations in the process of preparation of the sols or how much it has been run etc.

On the other hand, we note from the plots of the rear velocity of the slugs in Fig. 8 that the properties of the bentonite sols are prescribed carefully enough in so far as

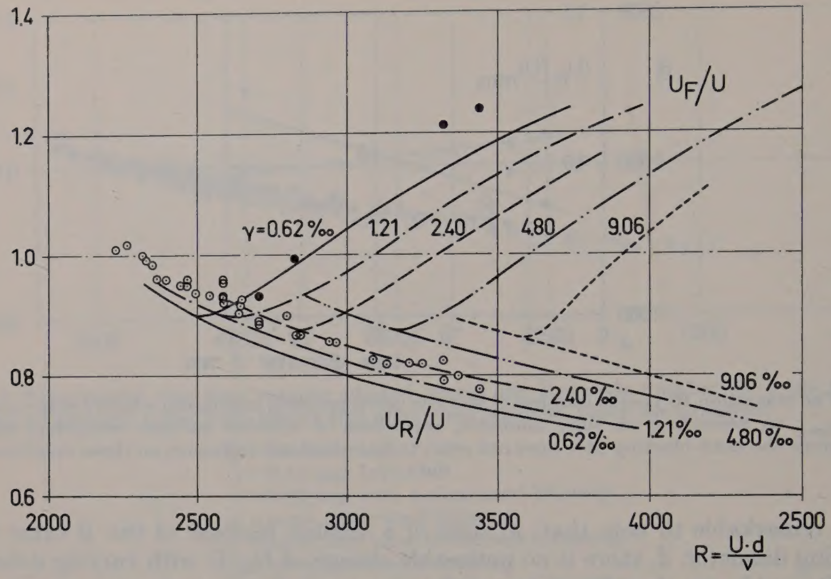

Fig. 8. Relative slug velocity for flows of the present 1.7 ‰ bentonite sol through the 23.72 mm tube before sand-blasting plotted within a diagram previously published (Lindgren 1959*c*, p. 107) for slugs in flows of bentonite suspensions of different concentration through the same tube. Note that the sol used in the present experiments is prepared from another parent suspension than that for the sols of the previous experiments.

The plot of the relative rear slug velocity for flows of the present sol (marked with open circles) appears to be about properly located in the diagram when compared with the previous experimental results. However, the present plot shows an extended region of existence of tubulent slugs towards lower Reynolds numbers and one also notes a shift of the relative front velocity of the slugs (marked with filled circles) towards higher values, implying a decrease of the R_k value (R_k = the Reynolds number at which the turbulent slugs begin to elongate during their downstream transport), in comparison with what was obtained for the flow of sols used previously.

concerns the reliability of the experimental determinations. Otherwise, the plot of the rear velocity of the slugs from the flow of the 1.7 ‰ sol should exhibit some improper shift relative to the plots for flows of the other sols shown in Fig. 8.

The front velocity of the slugs

The diagrams of Fig. 9 have been drawn in order to analyse the effect of the sand-blasting of the tubes. In all three diagrams the rear velocity relationship is the same as read from the mean plot of Fig. 6, limited by the proper transition Reynolds numbers $\underline{R}$ according to Fig. 7. (We note again that no systematic influence of the sand-blasting of the tubes could be traced with respect to the rear velocity data of the turbulent slugs.) The fully drawn line of the front velocity of the slugs, U_F, in each case respresents a "smoothed" curve adjoining the mean of the maximum values obtained for the front velocity of the slugs as read from the plots of Fig. 4 for the 10, 16 and 24 mm tubes. The dotted curves represent corresponding values of the slug front velocity as read from Fig. 5 after sand-blasting of the same tubes.

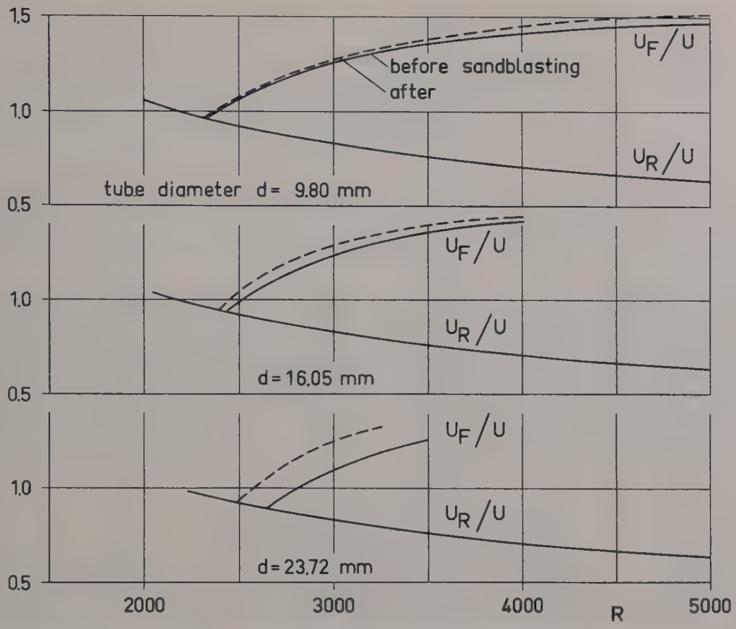

Fig. 9. Comparison of relative turbulent slug velocities before and after sand-blasting of the three larger tubes. We note an increase of the relative front velocity, U_F/U, and a decrease of the critical Reynolds number, R_k (the Reynolds number at which the slugs begin to elongate during their travel), with the increase of the surface roughness of the tubes, although the roughness elements still are of microscopic order of size. On the other hand the sand-blasting does not seem to have influenced the relative rear slug velocity, U_R/U, nor the transition Reynolds number $\underline{R}$ (the least Reynolds number at which self-preserving turbulent slugs ever appear). It is important to note that the difference in surface roughness before and after sand-blasting of the tubes is largest for the 24 mm tube and smallest for the 10 mm tube, a fact which seems to imply a correlation with respect to the experimental results.

Evidently, the sand-blasting has caused a decrease of the critical Reynolds number, R_k, with a corresponding shift of the U_F/U curves (respresenting the front velocities of the turbulent slugs) towards lower Reynolds numbers. That is, the U_F/U values for flows after sand-blasting of the tubes have become higher than those obtained at the same Reynolds numbers before sand-blasting.

In the first instant it appears quite surprising that the effects noted are most prominent for the largest of the three tubes, and that there is a subsequent decrease of the roughness influence with decreasing tube diameter. Here it must be noted that even if the influence of the sand-blasting of the 10 mm tube appears to be rather vague with respect to the development of the slug fronts, it nevertheless is a distinct effect when considering the uniformity of the data obtained, and that the effect observed in each case has been estimated along curves adjoining lowest possible limits of the evaluated data. Nor can the stronger influence of the sand-blasting on the larger tube be explained as due to some ageing effects of the flowing sol during the experiments, since they were always performed in the order of decreasing diameter both before and after sand-blasting of the tubes within an interval of about six months.

Another observation concerning the development of turbulent slug fronts should also be reviewed. The plot for the flows through the 6 mm tube, according to Fig. 4, shows two distinct branches, one with a higher and one with a lower relative velocity of the turbulent slug fronts, but both converging towards the same curve for higher R. An irregular spreading of the front velocities is often observed as in the plot for the flows through the 16 mm tube. Such an irregularity, however, although varying between two extreme limits, is not usually as systematic as that for the present 6 mm tube, where we note that the flow temperature for the lower branch varied between 15.9 and 16.7°C while the upper branch is connected to a temperature interval of 17.4 to 17.6°C.

The series of values belonging to the set 15.9–16.7°C is the only one belonging to such a low temperature interval since all other experiments reported here on slug front developments have been performed within a temperature interval of 17.4–20.0°C. No such temperature influence has been traced with respect to the rear velocity of the slugs. Not too much attention, however, should be paid to this pecularity since its verification is still pending.

Discussion of results obtained

Structural effects

For proper judgement of the observations noted we must consider some known features of the transition process. For instance, a turbulent slug develops its own characteristics, independent of the specific disturbances that initiated it, and in each case the formation of self-preserving turbulent slugs takes place at a lower limit of Reynolds numbers. $\underline{R}$, which seem to remain constant as soon as a certain least disturbance level is exceeded.

From the statements above it is evident that, since the sand-blasting did not influence the $\underline{R}$-values, the variation of $\underline{R}$ with tube diameter d cannot be due to variations of the relative roughness, k/d. This means that the formation of slugs from the inlet disturbances should not be influenced by microscopic surface roughness elements.

On the other hand, besides the relative roughness, only the flow velocity, i.e. the rate of shear, varies with the tube diameter for the same Reynolds number. The noted variation of $\underline{R}$ with d may be interpreted as a further verification of previous observations (Lindgren, 1959*c*, pp. 110–111) that structural properties of the liquid. influenced by the rate of shear and/or flow dimensions, could be relevant with respect to the process of formation of turbulent slugs. Whether this influence may be due specifically to the structural properties of the liquid forming the observed laminar wall-layer of the turbulent inlet flow (in the turbulent core the Reynolds stresses certainly could not let some structural differences remain between the flows through various tubes merely because of different diameter), or whether relaxation processes may possibly develop fast enough after decay of the small intense inlet eddies so that structural differences may occur between the bulk flows—this is a question that cannot be answered.

Similarly, the smaller $\underline{R}$-value obtained for flow through the 24 mm tube in the present experiments as compared to those of the former study in(spite of full correspondence between the plots considering concentration and viscosity of the sols as shown in Fig. 8) also indicates the presence of structural deficiences between the sols, without their bulk flow properties showing any measurable influence therefrom (see p. 457).

Also, the double branches of the slug front velocity obtained for the flow through the 6 mm tube may imply an indication of structural properties of the liquid influencing some characteristics of the turbulent slugs.

Boundary effects

The variation of the $\underline{R}$-value with tube diameter d is evidently to be connected only with the primary turbulent inlet flow and its breakdown, while the characteristics of the once formed turbulent slugs are not at all influenced by the primary flow and its disturbances. This is an important observation in view of the fact that the relationship between U_R/U and R appears to be fairly independent of variations of the tube diameter and microscopic changes of the surface texture. This could imply that structural properties of the liquid are not changed very much by different rates of shear while variations of concentration produce larger differences. This conclusion suggests that the increase of $\underline{R}$ with d could be due to different disturbance levels of the inlet flows for the various tubes rather than depending on an assumed influence of the shear stress on the structure of the flowing liquid. It would seem quite likely that the maximum disturbance level obtainable by the method used increases with increasing mean flow velocity. If we then remember that for the same Reynolds number the mean flow velocity decreases with increasing tube diameter, we receive perhaps a more simple explanation for the noted relationship between $\underline{R}$ and d. However, it is important not to forget that the given explanation does not apply to the corresponding variation of R_k noted.

We may also conclude, that, since the relative rear velocity of the slugs was not changed by the sand-blasting, differences between microscopic surface roughness do not noticeably influence the turbulent energy transposition, a fact which is in full accordance with generally accepted experiences of boundary-layer research. However, the effect noted that the sand-blasting increased the relative expansion velocity of the slugs, $(U_F-U_R)/U$, as shown in the diagrams of Fig. 9, gives an interesting contribution to effects observed, since this is not in agreement with accepted rules on this matter. Actually these observations would imply that microscopic surface roughness elements should influence the regeneration process of self-preserving turbulent eddies, but that they should not influence very much the energy balance of the self-preserving turbulent flow. These observations confirm experimental studies reported earlier, and the conclusion drawn above agrees with some ideas brought forth several years ago (Lindgren, 1957, pp. 161, 162).

The present findings appear rather confusing when we note the decrease of influence of sand-blasting with decreasing tube diameter. If the change in relative roughness were responsible for the differences between the expansion velocities it would appear natural to expect the largest effects for flow in the smallest tube. However, noting that the difference between the surface roughness before and after sand-blasting is larger for the 24 mm tube than for the 16 mm tube and still larger than for the 10 mm tube (compare Fig. 3), we may have a natural explanation for the variation of the recorded effects.

The fully developed turbulent flow

Should the findings reported and their interpretation above be valid, this would imply the existence of two types of turbulent flow as has earlier been hinted at repeatedly (Lindgren, 1957, pp. 59, 73, 145–146; 1959*a*, p. 117; 1959*b*, p. 504). One should be a type of decaying turbulent flow evidently possessing a laminar sub-layer in the strict sense. The other should be a type of self-preserving turbulent flow which possesses a "viscous" layer within which shear stresses dominate over Reynolds' stresses, and microscopic eddies shed from the surface of the bounding walls are taken care of by the turbulence production term which has its maximum value at the edge of the layer. The microscopic roughness eddies do not necessarily have to be identified with velocity gradients so high that they should be dissipated immediately by viscous action, as may readily be seen from the papers by Fage & Townend (1932) and Eckert & Soehnghen (1952). The turbulence production term thus may be large enough, even very close to the walls, to allow a continuous self-preserving process to develop.

In this connection mention should also be made of some important features of the turbulent slugs as previously reported. These are that at the rear of a turbulent slug, laminar flow enters it and becomes turbulent (Lindgren, 1957, p. 148); that the velocity distributions of the turbulent slugs and their laminar intervals are almost completely fully developed turbulent and laminar velocity profiles (Rotta, 1956); that under these circumstances, the entering laminar flow at the slug rear must have velocity components directed obliquely outwards, towards the bounding walls (Faxén, 1958, p. 236).

Conclusions

The reported findings indicate that:

(*a*) Shear stress may influence some structural properties of the flowing liquid crucial to the transition process; or the maximum disturbance level obtainable of the inlet flow, increases with increasing mean flow velocity, i.e. decreasing tube diameter, under the present inlet flow arrangements. Either of these two explanations could account for a noted increase of the transition Reynolds number $\underline{R}$ from about 1860 to 2220 (ca. 20 % increase) when the tube diameter d increased from 3.84 to 23.72 mm (ca 600 % increase). This is a verification of previous findings (Lindgren, 1959*c*) according to which the Reynolds number is but an approximate model law.

(*b*) The may appear structural differences of importance to the transition process in liquid flow, without any noticeable influence on its bulk flow properties being observed.

(*c*) Changes of microscopic roughness do not influence the breakdown of the primary inlet turbulent flow nor do they influence the rear propagation velocity of the turbulent slugs. This state of affairs may imply that the decaying inlet turbulent flow does possess a laminar sub-layer in the strict sense. Otherwise the microscopic roughness elements would have influenced the breakdown of the fully developed turbulent inlet flow. Nor does a change of miscroscopic surface roughness noticeably change the turbulent energy transposition of the turbulent slugs (otherwise a change of the roughness would have caused a change of the relative rear slug velocity).

(*d*) The change of diameter, i.e. rate of shear, has but a very small effect on the rear propagation velocity of the turbulent slugs, while a change of bentonite concentration produces large differences.

(*e*) Microscopic changes of the surface structure of the tube influence the relative expansion velocity of the turbulent slugs, indicating that the self-preservation of the turbulence of the slugs, and thus also of the turbulent flow in general, could perhaps be due to eddies shed from microscope surface roughness element at the flow boundaries and then taken care of by the turbulence generation term, which has its maximum value at the edge of the viscous layer.

(*f*) The critical Reynolds number, R_k, decreases with increasing (relative) roughness (of microscopic order of size) in accordance with a noted decrease of R_k with decreasing d (while the surface finish was about the same). However, we cannot exclude that also structural effects within the flowing liquid could account for the variation of R_k with d, while possible variations of the inlet flow disturbance level cannot be a relevant factor in this respect, even they may be so with respect to the variation of $\underline{R}$ with d (see (*a*)). Structural effects, however, scarcely could account for the noted decrease of R_k with increasing roughness, nor could any variations of the inlet flow disturbance level. This is another indication that the Reynolds number does not constitute a complete model law at least for liquid flow in tubes.

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